GPH 503: Homework 8

Question 1 (*10 pts*): Suppose $z = x + iy$ with $x, y \in \mathbb{R}$. Write the following in the form a+ib where $a, b \in \mathbb{R}$:

(a) $\frac{1}{3+i}$ (b) $\frac{z+\bar{z}}{1+\bar{z}}$ (c) $(1+i)^{100}$

Question 2 (*10 pts*): Sketch the regions of the complex plane

(a) $e < i(z+i) < \pi$ (b) $|z-i| \geq |z+1|$

Question 3 (***10 pts***): Find and plot all values of

(a) $(2-i)^{1/2}$ (b) $i^{1/5}$

Question 4 (*15 pts*): Consider the strip $1 < y \leq 2$ and $-\infty < x < \infty$ in the complex plane where $z = x + iy$.

(a) (*7 pts*) Find the image of the strip under the mapping $w = f(z) = z^2$.
(b) (*8 pts*) Find a mapping $w = f(z)$ that maps the strip exactly once to the whole w plane.

Question 5 (*5 pts*): Show that if $Re(z_1) > 0$, $Re(z_2) > 0$ then

$$Log(z_1 z_2) = Log(z_1) + Log(z_2)$$

Hint: I would like to remind that $Log(z)$ is the principal value of $log(z)$, so $-\pi < Arg(z_1) < \pi$, $-\pi < Arg(z_2) < \pi$.

Question 6 (*15 pts*): Evaluate and plot the following functions in the complex plane
Note: Do not forget to consider the multivalued nature of the functions.

(a) 2^i (b) $(\cos\ i)^{\mathrm{i}}$ (c) $log\left(\frac{1}{1+i}\right)$

Question 7 (*15 pts*): Identify the branch points and the number of branches for the following functions in the extended complex plane:

(a) $\cos\,(z^{1/2})$ (b) z^z

Question 8 (*20 pts*): Consider the function $w = f(z) = (z^3 + z^2 - 6z)^{1/2}$ in the extended complex plane.

(a) (*5 pts*) Identify the branch points
(b) (*15 pts*) Give a polar description of $f(z)$ and define a single branch that is continuous at $z = -1$ with $f(-1) = -\sqrt{6}$. Sketch (plot) the branch cuts on the complex plane.