

## GPH 503: Homework 7

**Question 1 (20pts):** Find the eigenvalues and eigenvectors of the following matrices. Plot the  $x$  vs  $Ax$  where  $x$  is a unit circle (similar to eigshow in Matlab). You don't have to plot them on the computer, sketches are fine. In which cases are the eigenvectors orthogonal?

(a)  $A = \begin{bmatrix} 6 & 16 \\ -1 & 4 \end{bmatrix}$       (b)  $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$       (c)  $A = \begin{bmatrix} 3 & -2 \\ -2 & 0 \end{bmatrix}$       (d)  $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$

**Question 2 (10 pts):** Find the eigenvalues and eigenvectors of the following matrices

(a)  $A = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$       (b)  $A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

**Question 3 (10 pts):** (a): Find the singular value decomposition of  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(b) Find the singular value decomposition of  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

**Question 4 (15 pts):** The transformation  $S$  takes the second derivative. Let  $1, x, x^2, x^3$  be the basis for  $\mathbf{V}$ . Let the basis for  $\mathbf{W}$  be  $1, x$ .

- (a) Write the 2 by 4 matrix  $B$  for the transformation  $S$ .  
 (b) Find the pseudoinverse  $B^+$ , of  $B$ . Calculate  $B^+B$  and  $BB^+$ . Are the results the identity matrix? Explain why or why not considering the nature of the transformation?

**Question 5 (10 pts):** Suppose  $T(v_1) = w_1 + w_2 + w_3$ ,  $T(v_2) = w_2 + w_3$  and  $T(v_3) = w_3$ .

- (a) Find the matrix  $A$  for the linear transformation  $T$  using these as basis vectors. What input vector  $v$  gives  $T(v) = w_1$ .  
 (b) Invert the matrix  $A$  for the transformation above. What is  $T^{-1}(w_1)$ ,  $T^{-1}(w_2)$  and  $T^{-1}(w_3)$ ?

**Question 6 (10):** Points consider a transformation represented by a matrix  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ . Diagonalize this matrix, determine the eigenvectors and explain what it does to a given vector (Rotation, or strain or reflection etc...).

**Question 7(10 pts)** Explain the vectors  $e = [1,0,0,0]$  and  $v=[1, -1, 1, -1]$  in the wavelet basis with  $w_1 = [1,1,1,1]$ ,  $w_2 = [1,1, -1, -1]$ ,  $w_3 = [1, -1, 0, 0]$ ,  $w_4 = [0, 0, 1, -1]$ .

**Question 8 (15):** Compute the SVD of following matrices and explain what it does to a given vector. Calculate  $A^+$  and also explain which basis vectors span the row space, which spans the null space, column space and left null space.

(a)  $A = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}$       (b)  $A = \begin{bmatrix} 3 & 3 \\ -1 & 1 \end{bmatrix}$       (c)  $A = [3 \ 4 \ 0]$