GPH 503 Homework Assignment 5

Least Squares Approximation

Question 1 (*10 pts*): For **b**={0,8,8,20} and **t**={0,1,3,4} set up $A^T A\hat{x} = A^T b$ and find the closest line C+Dt using least squares approximation. Plot the straight line and the data points in matlab (or on a graph paper) and calculate the total error . Check that error is perpendicular to both columns of A.

Question 2 (*5 pts*): Now change the data points to b={1,5,13,17} and solve directly for a straight line (Hint: Use elimination).

Question 3 (10 *pts*): Now write down $E = \|Ax - b\|$ sum of for squares for **t** and **b** in question 1. For example last term should be $E = (C + 4\mathrm{D} - 20)^2$. Sum to obtain the function E. Then set $\frac{\partial E}{\partial C}$ and $\frac{\partial E}{\partial D}$ Divide by 2 to obtain the normal equations $A^T A\hat{x} = A^T b$.

Question 4 (15 *pts*): This time, solve the question for the closest parabola $b = C + Dt + Et^2$ for the same points. Write down the unsolvable $Ax = b$. Set up the equations $A^T A\hat{x} = A^T b$. Now instead of 2, you have 3 free variables to fit the data. Solve the equations (in MATLAB, because it would be hard to take the inverse by hand).

Orthogonal Bases and Gram-Schmidt

Question 5 (15 *pts*): (a) (8 pts) Find orthonormal vectors $\mathbf{q_1}$ and $\boldsymbol{q_2}$ in the plane of **a**=(1,3,4,5,7) and **b**=(-6,6,8,0,8)

(b) (7 pts) Which vector in this plane is closest to (1,0,0,0,0).

Question 6 (15 pts) Find an orthonormal base for the column space of A, then compute the projection of b onto that column space for $A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$.

Determinants

Question 7 (10 pts) Compute the determinants of A and B. Are their columns independent.

(a) $A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 1 & 6 \\ 2 & 3 & 5 & 2 \\ 1 & 4 & 4 & 2 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Question 8 (15 pts)

(a) (5 pts) Find the area of the paralelogram with edges $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

(b) (5 pts) Find the area of the triangle with sides v , w and $v + w$. Draw it.

(c) (5 pts) Find the area of the triangle with sides v , w and $v - w$. Draw it.

Question 9 (5 pts) A box has edges from (0,0,0) to (3,1,1) and (1,3,1) and (1,1,3). Find its volume.