GPH 503 Homework Assignment 4

The Complete Solution to $Ax=b$

Question 1 (*25 pts*): For the following matrices A and b, find the complete solution to $Ax=b$. Follow the steps below to obtain the complete solution.

1. Augment A and b and perform row elimination operations to bring the system into reduced row echelon form. That means you need to do elimination top to bottom and then bottom to top and make sure pivots are the only nonzero elements of that column. Also divide rows by the pivot to make all pivots 1. Call this new matrix R.
2. Determine m, n, rank r, the dimensions of the row subspace in $\boldsymbol{R}^n$ and column subspace in $\boldsymbol{R}^m$. Determine also the dimen sions of the null space and the left null space (i.e. null space of transpose of A)
3. Then solve $Rx=0$ by making one of the free variables 1 while keeping other(s) 0. Determine the vectors that span the null space. The number of vectors that span the nullspace should be (n-r). Check that $As=0$ where each *s* is one of the special vectors that span the nullspace. Because null space should be orthogonal to row space $As=0$ should hold.
4. Then determine a particular solution $Rx=b'$ where b' is the b after the elimination to bring the system from A to R.
5. Finally write the solution to Ax=b.

(a) (*10 pts*) $A=\begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}$ $b=\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$ (b) (*8 pts*) $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ $b=\begin{bmatrix} 4 \\ 4 \end{bmatrix}$

(c) (*7 pts*) $A=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ $b=\begin{bmatrix} 8 \\ 22 \\ 22 \end{bmatrix}$

Independence, Basis and Dimension

Question 2 (*20 pts*): Find a basis for each of the subspaces of $\mathbf{R}^3$.

(a) (5 *pts*) All vectors whose components are equal.

(b) (5 *pts*) All vectors whose components add to zero

(c) (*10 pts*) All vectors that are perpendicular to (1,1,0) and (1,0,1)

Orthogonality

Projection onto a line

Question 3 (15 pts) Project vector **b** onto the line through **a**. Check that **e** is perpendicular to **a**.

(a) $a=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $b=\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ (b) $a=\begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$ $b=\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

Question 4 (10 pts) Project the vector **b**=(1,1) on to the lines $\mathbf{a_1}$=(1,0) and $\mathbf{a_2}$=(1,2). Draw the projections $\mathbf{p_1}$ and $\mathbf{p_2}$ and add $\mathbf{p_1}+\mathbf{p_2}$. Is $\mathbf{b}=\mathbf{p_1}+\mathbf{p_2}$? Why or Why not?

Projection on to a subspace

Question 5 (20 pts) Project b onto column space of A. Calculate $\hat{x}$, projected vector $\vec{p}$ and projection matrix ***P***.

(a) $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $b=\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ (b) $a=\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ $b=\begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$ (c) $a=\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ $b=\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Question 6 (*10 pts*) For a peojection matrix $P=A(A^T A)^{-1} A^T$ show that $P^2=PP$ is equal to P.