Name____ *due* 10/10/2011 in class

GPH 503: Homework 2

Inverse of a Matrix

Question1 (*30 pts*): Find the inverse of the following matrices using Gauss-Jordan Elimination.

a) Triangular Pascal Matrix $\mathbf{A}=\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 2 & 1 & 0\\ 1 & 3 & 3 & 1\end{bmatrix}$

b) Permutation Matrix $P=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$. Think of solving the same matrix without Gauss-Jordan elimination. State what this matrix does when it multiplies another matrix from left. *i.e.* $\boldsymbol{PA}$. And how you can undo it such that $\boldsymbol{P^{-1}}(\boldsymbol{PA})=\boldsymbol{A}$ for an arbitrary matrix A.

c) $\mathbf{R}=\begin{bmatrix}0 & 0 & 0 & 2\\ 0 & 0 & 3 & 0\\ 0 & 4 & 0 & 0\\ 5 & 0 & 0 & 0\end{bmatrix}$

d) $\mathbf{B}=\begin{bmatrix}3 & 2 & 0 & 0\\ 4 & 3 & 0 & 0\\ 0 & 0 & 6 & 5\\ 5 & 0 & 7 & 6\end{bmatrix}$

Question 2 (*10 pts*): For $[A]_{3\times3}$ matrix such that column3 = column 1 + column2; show that no inverse exists. (*Hint:* Find a nonzero vector $\boldsymbol{x}$ such that $\boldsymbol{Ax}=\mathbf{0}$**;** which is sufficient that no inverse exists for A)

Question 3 (*10 pts*): If the product $\boldsymbol{C}=\boldsymbol{AB}$ is invertible, (***A*** and ***B*** are square) then $\boldsymbol{A}$ itself is invertable. Find a formula for $\boldsymbol{A}^{-1}$ in terms of $\boldsymbol{C}^{-1}$ and $\boldsymbol{B}$.

LU Decomposition

Question4 (*25 pts*): Find the LU decomposition and solve first **Ly**=**b**, then ***Ux***=***y***, to obtain solutions for the following systems of equations.

a) $\begin{aligned}x+y&=5\\ x+2y&=7\end{aligned}$

b) $\begin{aligned}x+\ y+\ z&=5\\ x+2y+3z&=7\\ x+\ 3y+\ 6\,z&=11\end{aligned}$

c) $A=\begin{bmatrix}1 & 0 & 1\\ 2 & 2 & 2\\ 3 & 4 & 5\end{bmatrix}$ $b=\begin{bmatrix}3\\ 4\\ 7\end{bmatrix}$

Question 5 (10 pts): For the tridiagonal matrix ***T***, find the ***LU*** decomposition. Observe which elements are zero in ***L*** and ***U*** and compare them to ***T***. Explain how you could take advantage of the sparseness of T if you were to write a program.

$$T=\begin{bmatrix}1 & 2 & 0 & 0\\ 2 & 3 & 1 & 0\\ 0 & 1 & 2 & 3\\ 0 & 0 & 3 & 4\end{bmatrix}$$

Question 6 (15 pts): Compute the LU decomposition for the symmetric matrix A and nonsymmetric matrix B. Find 3 conditions for A and 4 conditions for B so that they have 4 pivots.

$$A=\begin{bmatrix}a & a & a & a\\ a & b & b & b\\ a & b & 2 & 3\\ a & b & 3 & 4\end{bmatrix}; \quad B=\begin{bmatrix}a & r & r & r\\ a & b & s & s\\ a & b & c & t\\ a & b & c & d\end{bmatrix}$$