

GPH 503: Homework 1

Question 1 (20 pts): Plot the row and column picture of the following systems of linear equations (in Matlab) and determine the solutions or show that no solution exists. The row picture may correspond to a line or a plane for each equation so you need to plot those lines or planes.

- a) $x + y = 1$; $x + 4y = 7$;
 b) $-3x + y = 1$; $6x - 2y = -3$;
 c) $x - y + 2z = 9$; $-2x + 3y + z = -4$; $-y + z = 4$. For this system of linear equations, also determine the lines

Question 2 (20 pts): Perform the following matrix operations

- a) $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix}$ $\mathbf{b} = [3 \quad -1 \quad -1]$. Calculate \mathbf{ab} .
 b) $\mathbf{a} = [2 \quad -1 \quad 1]$; $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 3 \\ 3 & 4 & 1 \end{bmatrix}$. Calculate \mathbf{aB} and \mathbf{Ba}^T .
 c) $\mathbf{A} = \begin{bmatrix} 5 & 5 & -4 \\ -4 & 0 & -1 \\ 5 & 3 & 5 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 3 \\ 3 & 4 & 1 \end{bmatrix}$ Calculate $(\mathbf{AB})^T$ and $\mathbf{B}^T\mathbf{A}^T$.
 d) $\mathbf{A} = \begin{bmatrix} 5 & 5 & -4 \\ -4 & 0 & -1 \\ 5 & 3 & 5 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 3 \\ 3 & 4 & 1 \end{bmatrix}$ $\mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ Calculate $(\mathbf{AB})\mathbf{c}$ and $\mathbf{A}(\mathbf{Bc})$. Are they same or different? Comment.

Question 3 (20 pts): Find the pivots, apply Gaussian elimination to reduce to system to upper triangular form and use back substitution to solve following systems of linear equations.

- a) $2x + 3y + z = 8$
 $4x + 7y + 5z = 2$
 $-2y + z = 0$
- b) $2x - 3y = 8$
 $4x - 5y + z = 7$
 $2x - y - 3z = 5$
- c) $x + y + 3z = 8$
 $4x - 5y + z = 7$
 $2x - y - 3z = 5$

Question 4 (20 pts): Bring the following systems of equation into augmented matrix form and use elimination matrices to bring them to upper triangular form and solve them. If there are no solutions, state it, if there are multiple solutions, find the solutions in terms of a variable. Make sure you determine and explicitly write all the elimination matrices.

- a) $2x - 3y = 8$
 $4x - 6y + z = 7$
 $2x - y - 3z = 5$
- b) $2x - 3y = 8$
 $4x - 8y + 3z = 7$
 $2x - y - 3z = 5$
- c) $2x + y = 0$
 $x + 2y + z = 0$
 $y + 2z + t = 0$
 $z + 2t = 5$

Question 5 (20 pts): For which three numbers a will elimination fail to give three pivots for the following

matrix \mathbf{A} ? $\mathbf{A} = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$